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Effect of Surface Textures on the Performance of Off-set halves Porous Journal Bearing for Couple Stress Fluids

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Abstract : Tribology is advance branches of science, deals the activity that occur between two body's surfaces with minor or major relative motion known as friction and wear. Roughness has occurred due to machining defect such as deterministic roughness, surface texture can be form of many shapes. This paper contained the three types of surfaces texture such as sinusoidal, half wave and full wave with both type longitudinal and transverse for off-set halves bearing profile purpose to enhance the bearing performance of bearing. Porous bearing considered as a self-lubricated bearing due to their low friction and oil absorption phenomena.

Keyword: Lubricant additives, Off-set halves journal bearing, roughness, finite difference method and porous bearing.

1. INTRODUCTION

The study of surface textured on the hydrodynamic porous journal bearings with couple stress fluids is a subject of growing interest to enhanced value of pressure profiles to reducing the damping effect and make stable to the journal as compare to load carrying capacity. Off-set halves journal bearing is widely used for their stability, generating two lobs circular bearing. Surface texture is inherent to the process used in their manufacture.

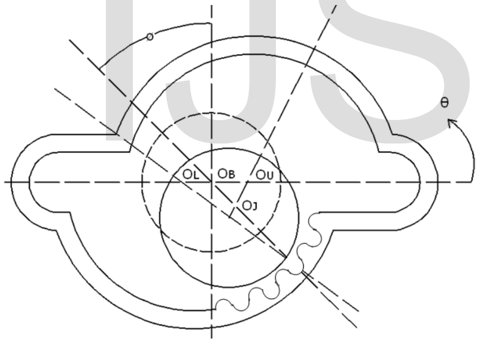

Fig. 1

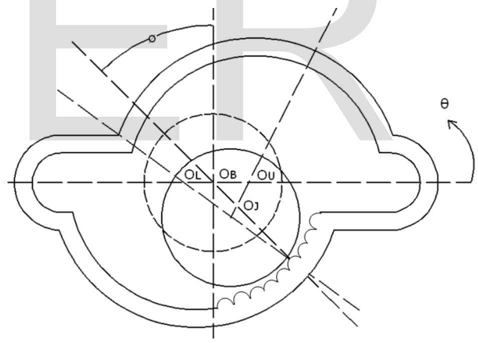

Fig. 2

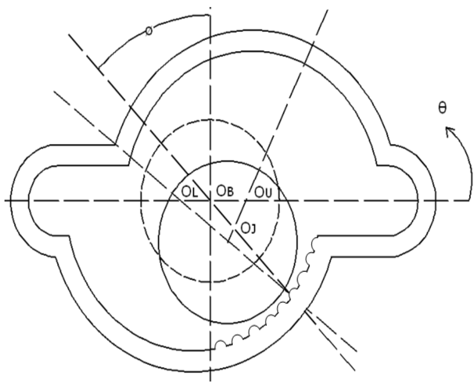

Fig. 3

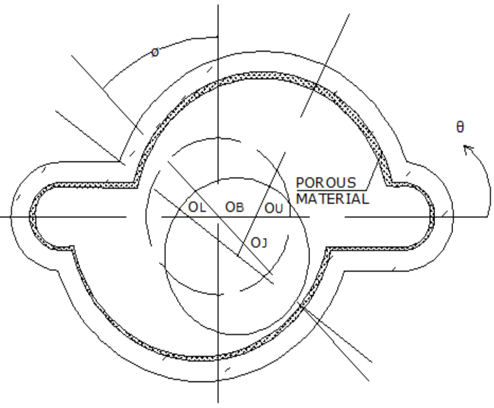

Fig. 4

Off-set halves journal bearings are widely subject to use in industry because of their simplicity and stability. Porous material has a specialty to absorb the lubricant which came in their contact by filling oil in the pores, now a day porous bearing (Fig. 4) widely used for reducing friction force and known wave textured respectively. Couple stress fluids used as a Non-Newtonian fluid, which is Newtonian fluids with some additive. Additive used in this change the chemical composition of oil and increases the shearing capability of oil present between journal and bearing. Aggressive research is being carried out worldwide to explore the performance and reduce the wear of bearing with the improved characteristics.

Newkirk and Taylor experimental reported a new kind of self-excited rotor dynamic instability in hydrodynamic type journal bearings. They observed that during this instability of the rotor orbits in its bearing at a frequency approximately half of the rotor speed. Allaire et al. [1] presented the numerical analysis of four types of bearing profiles (off-set halves, two lobe, three lobe and four lobes) and computed that off-set halves bearing exhibited minimum amount of vibration. Pinkus and Lynn [2] predicted the power loss of three lobes journal bearing by considering symmetrical and unsymmetrical function. Further the possibility of displacing the lobe centers of two lobe journals bearing with respect to mid-radius analyses [3].

Black and Murry [4] studied the bearing performance characteristic for laminar and turbulence regimes of different bearing geometry. When multi lobe bearing configurations are consider, the load magnitude and direction are fully dependent on bearing characteristic and not directly calculated. The development of pressure Cameron first investigated the Reynolds equation for porous bearing and effects on bearing parameter after then Rouleau carried out the theoretical study for porous journal bearings. Famous Darcy's equation was used for above. Analytic solution is given by Murti [11-12] for long and short porous journal bearing has given and used modified Reynolds's equation, Galerkin method and continuity equations author mention that load carrying capacity reduces friction force also reduces by increasing permeability factor. Author also investigated that wall thickness effects as same fashion, at some level of film thickness porous bearing work as a self-lubricated bearing.

Reason and Dyer [13] solve the modified Reynolds equation for the hydrodynamic lubrication of finite Porous journal bearings by using Darcy's equation. A numerical solution occurs by finite difference method and assumes no slip condition. Cusano [14] computed and compare the load capacity obtained by using the finite bearing for varying eccentricity ratio and permeability factor then after that the solution considers the curvature of the bearing wall as self-lubricant bearing. Figure (1-3) show the off-set halves journal bearing with sinusoidal wave, full wave and half

profile for four lobe bearing are predicted by Flack et al. [5] and it is observed that the trend of pressure with respect to rotational speed are same for half Sommerfeld condition but varies for Reynolds boundary conditions. Singh and Gupta [6] computed the stability limits of elliptical journal bearings for supporting flexible rotor. The stability of submerged four lobe oil journal bearing under dynamic load condition is analyses by Pai and Mayumdar [7] on using a non-linear transient method to predict the journal center trajectory. Mehta and Rattan [8] reported that multi lobe bearing is consider to be more stable than ordinary circular bearing in industries and performance of three lobe pressure dam bearing is far supporting to that of an ordinary three lobe bearing. The tilted pad and non-circular journal bearing are generally used for high speed and low load because their advantage of stabilization [9].

Kango and Sharma [10] investigated the combined influence of surface texture, using sinusoidal, positive full and half wave roughness (transverse and longitudinal roughness) and non-Newtonian lubricants, obeying power law model on circular journal bearing. The modified Reynolds equation is solved numerically through finite difference approach for analysis of texture and non-Newtonian effects on bearing performance characteristics.

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observed by Reason and Sew [15]. Modified Brinkman-extended Darcy model and Stokes equations computed by Elsharkawy and Guedouar [16] for hydrodynamic lubrication of porous journal bearings and the viscous shear effects of the Brinkman model signify an improvement in the steady characteristics of the system for long flexible porous journal bearings by Lin et al. [17]. Effect of surface roughness on the squeeze film characteristics for long porous partial journal bearings with considering couple stress fluids as a lubricant, adopting Stokes micro continuum theory computed by Naduvinamani et al. [18] The finite modified Reynolds equation was derived from the Stokes constitutive equations for couple stress fluids and solved numerically by using the finite difference technique and concluded that the effect of couple stresses was to increase the squeeze film pressure, load carrying capacity by Naduvinamani and Patil [19].

2. MATHEMATICAL MODELING

2.1. Film Thickness Equation

Film thickness equations used in the analysis of non-circular journal bearing profiles are given in the following subsections.

2.1.1. Off-Set Halves Journal Bearing

The file thickness equations for the two lobes of off-set halves journal bearing [Sehgal R et al 2000] are given as:

$$h = c_m\left[\left(\frac{1+\delta}{2\delta}\right)+\left(\frac{1-\delta}{2\delta}\right)\cos\theta-\varepsilon\sin\left(\phi-\theta\right)\right] \quad (0<\theta<180) \qquad (1)$$

$$h = c_m\left[\left(\frac{1+\delta}{2\delta}\right)-\left(\frac{1-\delta}{2\delta}\right)\cos\theta-\varepsilon\sin\left(\phi-\theta\right)\right] \quad (180<\theta<360) \qquad (2)$$

θ =Angular position

It is assumed that the film thickness does not varying along the length (z-direction).

2.1.2. Wave textured Journal Bearing

The modification factor to account for textured in bearing surfaces and the modified film thickness equation are given as [10]:

$$\delta_{sinusoidal} = A_m \sin(K) \qquad (3)$$

$$\delta_{full\,wave} = \left(\frac{2A_m}{\pi}\right) - \left(\frac{4A_m}{\pi}\right)\left(\sum_{q=2,4,6....}\frac{\cos(qK)}{q^2-1}\right) \qquad (4)$$

$$\delta_{half\,wave} = \left(\frac{A_m}{\pi}\right) - \left(\frac{2A_m}{\pi}\right)\left(\sum_{q=2,4,6....}\frac{\cos(qK)}{q^2-1}\right) + \frac{A_m\sin(K)}{2} \qquad (5)$$

Where $K = \frac{\pi R\theta}{w}$ transverse wave

$K = \frac{\pi Z}{W}$ longitudinal wave

W=wave length = $\frac{\text{length of asperity section}}{\text{number of asperity}}$ A_m = Amplitude

wave texture surface film thickness replace by : $h_{final} = h - \delta_s$

2.2. Reynolds Equation:

The equation that governs the generation of pressure in lubricating films is known as the Reynolds equation. Reynolds (1886) first derived the equation, and it forms the foundation of hydrodynamic lubrication analysis. Reynolds equation is derived by the combination of the Navier-stokes equation and Continuity equation by using some assumptions. [20-21]

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z} = 0 \qquad (6)$$

Conservation of momentum (Navier-Stokes equation):

$$\rho\frac{DV}{Dt} = F_B - \nabla.P + \nabla.\tau_{i\,j}$$

Where V is velocity vector, ρ is density, τ is shear stress and P is pressure.

Using assumption, Continuity equation can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0$$

The Navier Stokes equation reduces to:

$$\frac{\partial \tau_x}{\partial x} = \frac{\partial P}{\partial x}$$
$$\frac{\partial \tau_z}{\partial z} = \frac{\partial P}{\partial z}$$

Using above Equations we get the Reynolds Equation, which is given below:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial z}\right) = 6U\frac{\partial h}{\partial x} + 12(v_H - v_0) \quad (7)$$

Where h is the film thickness, VO is velocity of fluid flowing in and VH is the velocity of fluid flowing out.

2.3. Reynolds equation for Porous Journal Bearings in case of Newtonian fluids[22]

The porous bearing length in the axial direction is L and nominal film thickness in the radial direction is h. The origin is taken on the oil/sinter interface. The sinter is of thickness H, extending down to y = -H, and has a permeability ϕ .

The journal moves at a surface velocity U and the oil film thickness as given in equation 1and equation 2. The flow through the sinter is governed by Darcy's law which can be given mathematically as

$$v = -\frac{\partial p}{\partial y}\frac{\phi}{\eta}$$

Where v is the velocity of flow across unit area and can be taken as q, ϕ is the permeability and η is the viscosity. There is a negative sign as the flow is in the direction of decreasing pressure.

The equation of continuity of flow is given as:

$$\frac{\partial}{\partial x}q_x + \frac{\partial}{\partial y}q_y\frac{\partial}{\partial z}q_z = 0 \quad (8)$$

By solving above equations The Reynolds equation in three dimensions is given by

$$\frac{\partial}{\partial x}(h^3\frac{\partial p}{\partial x}) + \frac{\partial}{\partial z}(h^3\frac{\partial p}{\partial z}) = 6\eta[U\frac{\partial h}{\partial x} + 2(v_H - v_0)] \quad (9)$$

As V_H is the velocity of fluid flowing out of the porous region, so it becomes zero and V_o is velocity of fluid flowing inside the porous region, so

$$V_H = 0 \;\&\; V_0 = -\frac{\partial p}{\partial y}\frac{\phi}{\eta}$$

$$\frac{\partial}{\partial x}(h^3\frac{\partial p}{\partial x}) + \frac{\partial}{\partial z}(h^3\frac{\partial p}{\partial z}) = 6\eta[U\frac{\partial h}{\partial x} + 2\left(\frac{\partial p}{\partial y}\right)_{y=0}\frac{\phi}{\eta}] \quad (10)$$

$$\frac{\partial^2 p}{\partial y^2} = -\left[\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right] = -K$$

By integrating the equation $\frac{\partial p}{\partial y} = -Ky + C_1$

Using boundary conditions, $y = -H, \frac{\partial p}{\partial y} = 0$

$$C_1 = -KH$$

Hence $\frac{\partial p}{\partial y} = -K(y + H)$

$$\left.\frac{\partial p}{\partial y}\right|_{y=0} = -\left[\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right]$$

Now above equation becomes as

$$\frac{\partial}{\partial x}(h^3\frac{\partial p}{\partial x}) + \frac{\partial}{\partial z}(h^3\frac{\partial p}{\partial z}) = 6\eta[U\frac{\partial h}{\partial x} - 2(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2})\frac{\phi H}{\eta}] \quad (11)$$

2.4. Reynolds Equation for Porous Journal Bearing with Couple stress fluids [22]:

Stokes theory is the simplest generalization of the classical theory of fluids, which allows for the polar effects such as the presence of non-symmetric stress tensor, the couple stresses and the body couples. Based on Stokes equation, the Reynolds equation with couple stress fluids is obtained, which is known as Non-Newtonian type Reynolds equation.
The continuity and momentum equations governing the motion of the lubricant in the absence of body forces and body couples are

$$\nabla . V = 0$$

$$\rho \frac{DV}{Dt} = -\nabla p + \mu \nabla^2 V - \eta \nabla^4 V \quad (12)$$

The pressure gradients in circumferential and axial directions are given as:

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u}{\partial z^2} - \mu \frac{\partial^4 u}{\partial z^4}$$

$$\frac{\partial p}{\partial z} = \eta \frac{\partial^2 w}{\partial z^2} - \mu \frac{\partial^4 w}{\partial z^4}$$

Using the complete solution of above equations by finding the complementary function and integral, the Reynolds equation for journal bearing with couple stress fluids is given as

$$\frac{\partial}{\partial x}\left(g(h,l)\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(g(h,l)\frac{\partial p}{\partial z}\right) = 6\eta U \frac{\partial h}{\partial x} \quad (13)$$

Where $g(h,l) = h^3 - 12l^2\left[h - 2l\tanh\left(\frac{h}{2l}\right)\right]$

And $l = \sqrt{\frac{\mu}{\eta}}$

Where l is the couple stress parameter, μ is the coefficient of viscosity and η is material constant responsible for couple stress effects.
The generalized Reynolds equation for porous journal bearing with couple stress fluids, From above equation Reynolds equation for porous journal bearing in case of Non-Newtonian fluids. With the help of Stokes equation and Darcy's Equation while considering the effects of couple stress fluids in porous journal bearing, the required equation is given below:

$$\frac{\partial}{\partial x}\left(g(h,l)\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(g(h,l)\frac{\partial p}{\partial z}\right) = 6\eta\left[U \frac{\partial h}{\partial x} - 2\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right)\frac{\phi H}{\eta}\right] \quad (14)$$

2.5. Bearing Performance Parameters

Expressions used to determine bearing performance parameters such as load carrying capacity & friction force are discussed in the following sub section.

2.5.1 Load Capacity

The mean pressure distribution is determined by the integration of above pressure equation. The component of load capacity per unit width are then determined in the usual manner by further integrations.

$$W_x = \int\int P \sin(\theta)\, r\partial\theta\, r\partial z \quad (15)$$

$$W_z = \int\int P \cos(\theta)\, r\partial\theta\, r\partial z \quad (16)$$

2.5.2 Friction Force

The Shear stress on the moving surfaces in the form of roughness parameter is calculated as:

$$F = \frac{1}{v}\int_0^v \int_0^l \tau \quad r\partial\theta\, \partial z$$

2.6. Computational process and Finite difference discretization for journal bearing:

The theoretical solutions of modified Reynolds equations by considering Newtonian and Couple stress fluids are adopted for off-set halves journal bearings. The number of nodes in x & z- directions of the bearing has been taken as 100, 50 respectively.

P = ((A1+A2)/ (2*A7)) + ((A4*A5)/ (2*A3*A7))-((A8*A6)/ (2*A3*A7))

3. RESULTS AND DISCUSSION:

Obtained results are discussed in this section. Considered general input parameters are coupled in below [10].

Diameter, D	0.1m
Length/Diameter, L/D	1
Radial clearance, C	200μm
Minimum clearance, C_m	120 μm
Speed, N	3000 RPM
Eccentricity ratio, ε	0.6
Viscosity coefficient, η	0.0750 pa s
Number of wave asperity	10
Asperity Amplitude A_m	7.5 μm
Texture Angle for off-set halve	123^0
Asperity wavelength (W)	Angle of texture x radius / 10
Couple stress parameter (l)	0.2
Permeability factor	0.001-0.1

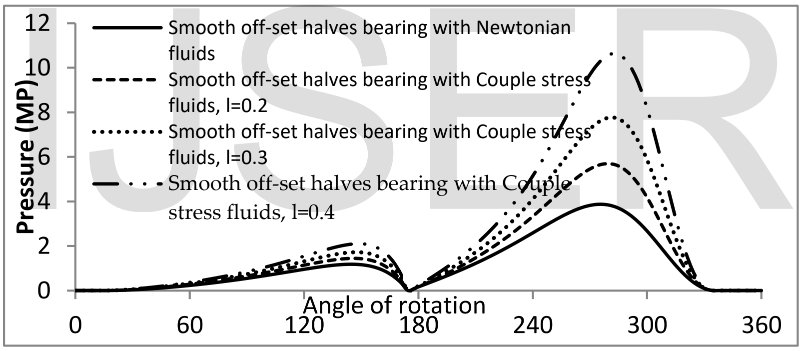

Fig.5

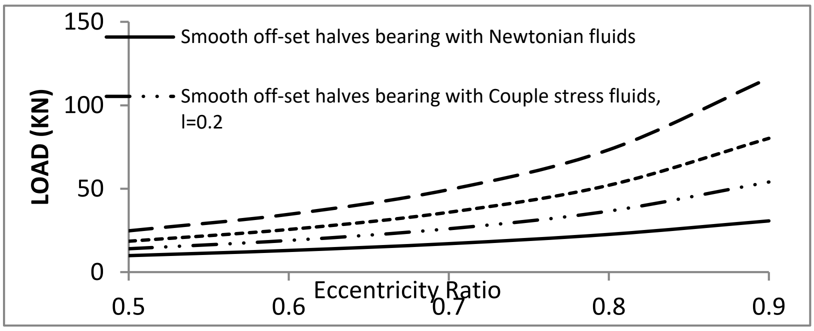

Fig.6

Figure 5 illustrate the pressure variation with respect to circumferential angle for Newtonian and Non-Newtonian (couple stress) fluids. Due to using couple stress fluid shearing in inner surface of lubricant is increase and by these phenomena high pressure is generated.

Result represented that as couple stress fluid considered the high pressure generated in lubricant. By increment of pressure the bearing can perform for high load carrying capacity as shown in figure 6. By increment of little character stick length (couple stress) major load carrying capacity can be achieved.

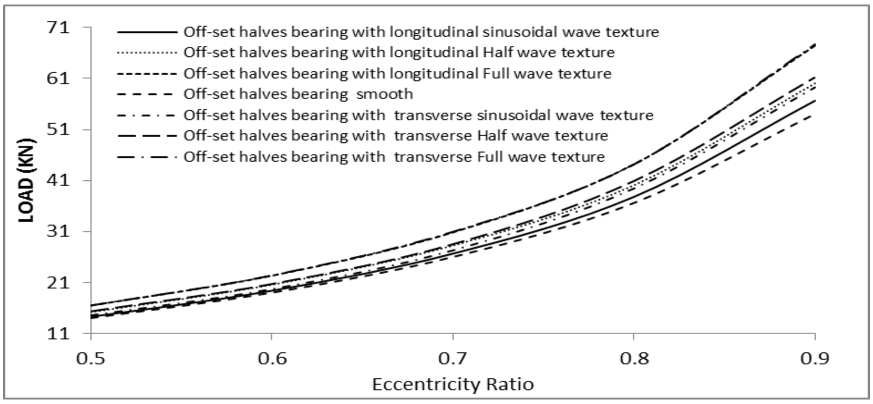

Fig.

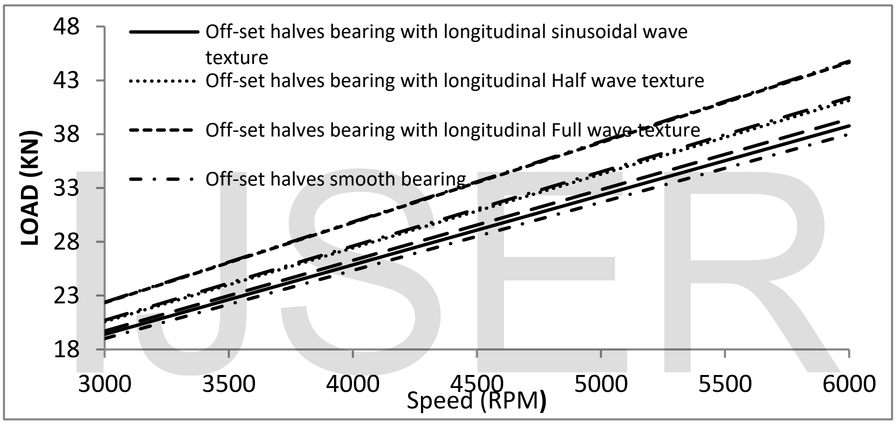

Fig.8

Figure 7 and figure 8 illustrate the comparative effect of different types of surface texture such as sinusoidal wave, full wave and half wave with longitudinal & transverse type wave texture with increasing eccentricity ratio and journal speed. It is observed that surface texture influence positively due to decreasing in minimum film thickness and full wave texture carries higher load carrying capacity as compare to half wave and sinusoidal wave.

Transverse wave texture gives better load carrying capacity as compare to longitudinal wave texture. As earlier discussed full wave has higher load carrying capacity than half wave, as same time by comparison of longitudinal and transverse half & full wave, transverse half wave give better result over longitudinal full wave texture. It is also observed that as eccentricity ratio and journal speed influenced load carrying positively.

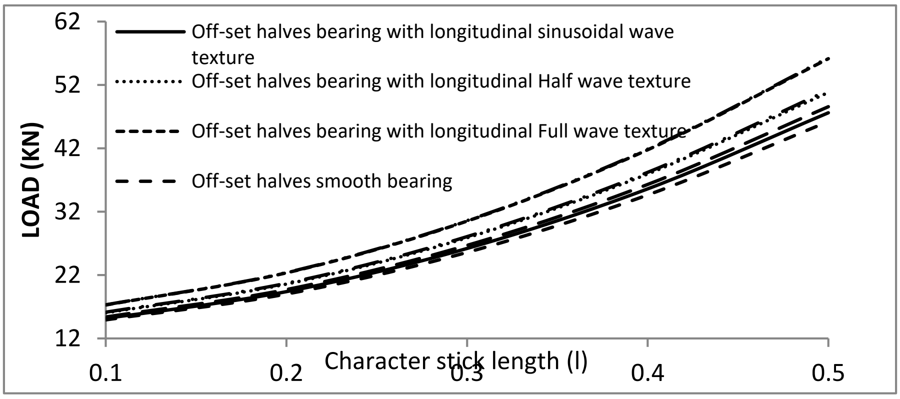

Fig.9

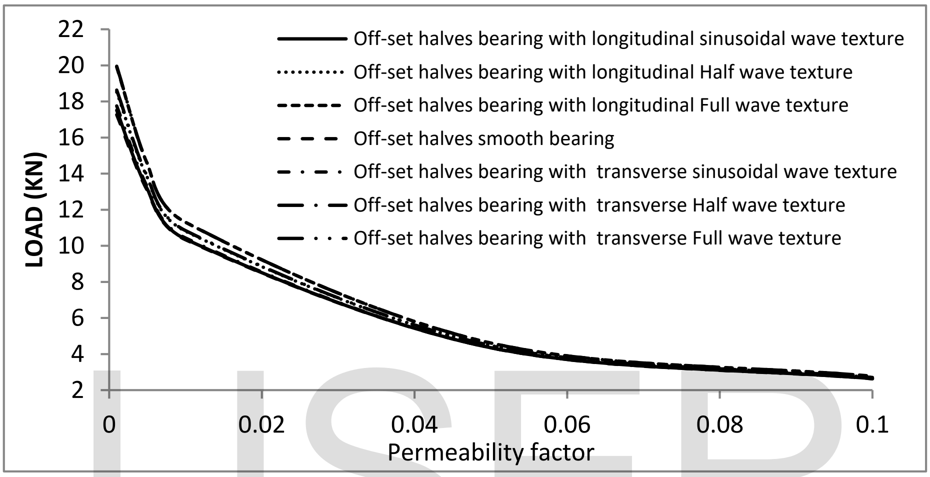

Fig.10

Couple stress fluids is a non-Newtonian fluids which is prepared by adding some types of chemical additive. Figure 9 represented that as considering couple stress parameter, load carrying increases for all types of surface texture due to high shearing effect.

Figure 10 illustrate that porosity or permeability factor affect the load carrying capacity, as permeability factor increases the load carrying capacity is decreases due to increment of film thickness at some or full region.

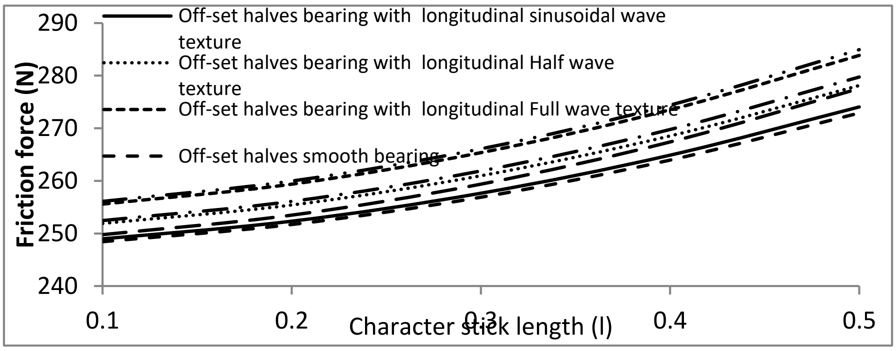

Fig.11

Couple stress fluids give high load carrying capacity due to high shearing effect in lubricants; by this type high shearing, friction force is increases with increasing couple stress character stick length, and increment in friction force is observed for all types of surface texture shown in figure 11.

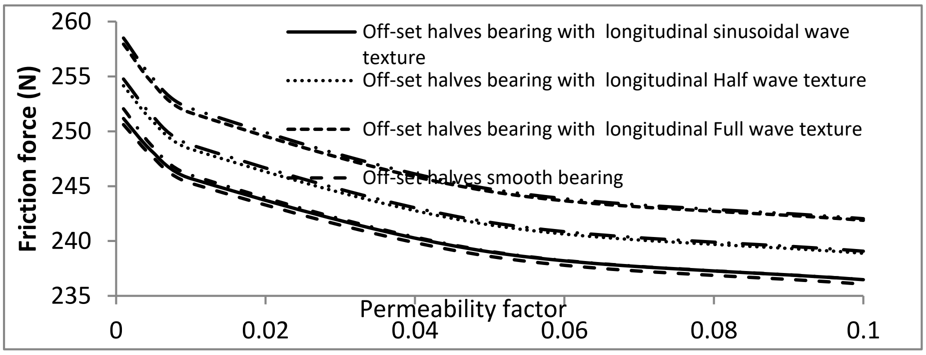

Fig.12

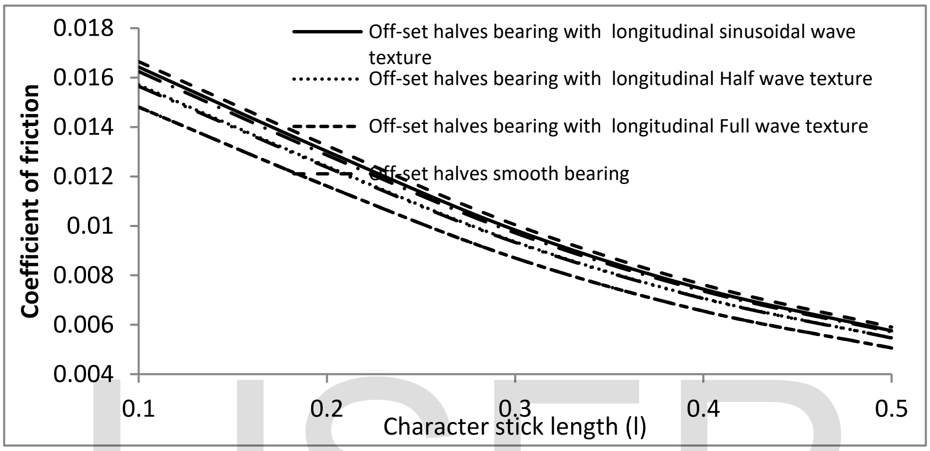

Fig.13

Figure 12 illustrate that by using porous bearing material friction force is decreases with increasing permeability factor or porosity. Porous materials are a self-lubricating material so due to this specialty they have ability to absorb the oil lubricant, and by this friction force reduces.

On using couple stress fluids enhance value of load carrying capacity permitted simultaneously increases in friction force which is not avoidable for journal bearing or slider bearing case. Figure 13 represented that as couple stress fluids parameter increases coefficient of friction decreases, coefficient of friction is totally relative to load carrying capacity and friction forces, and according to this method we got major increments in load carrying capacity with minor changes of friction forces.

4. CONCLUSION

Based on present investigation for off-set halves journal bearing with considering Newtonian and non-Newtonian fluids (couple stress fluids) and different types of surface texture, it has been carried out that load carrying capacity effect positively with proportional of couple stress fluids parameter and surface texture. It's also observed that porous bearing considering as a self-lubricated bearing due to their less friction forces, and these phenomena increases bearing life.

NOMENCLATURE

SYMBOLS	DESCRIPTION
A	Amplitude of asperity
c	Radial clearance
D	Bearing Diameter
E	Eccentricity
F	Friction force
H	Wall thickness of porous bearing

h	Film thickness
l	Couple stress parameter
L	Length of bearing
N	Shaft speed
P	Bearing pressure
r	Radius of shaft
U	Lubricant velocity in x-direction
W	Load carrying capacity
w	Wavelength of asperity
τ	Shear stress
ρ	Fluid density
Ø	Attitude angle
θ	Angular direction
ε	Eccentricity ratio
η	Lubricant viscosity
δs	Surface roughness variation
ϕ	Permeability parameter
μ	Material constant for the couple stress parameter

Appendix: A

$AA_1 = (h)/ (2*l)$

$G = ((h)^{3})-(12*l^{2}*h) + ((24*l^{3})*\text{Tanh}(AA_1))$

$A_1 = (P_{(i+1,j)}+P_{(i-1,j)})/(\Delta x)^{2}$

$A_2 = (P_{(i,j+1)}+P_{(i,j-1)})/(\Delta z)^{2}$

$A_3 = ((G_{(i,j)})+12*H*\psi);$

$A_4 = (G_{(i+1,j)}-G_{(i-1,j)})/(2*\Delta x)$

$A_5 = (P_{(i+1,j)}-P_{(i-1,j)})/(2*\Delta x)$

$A_6 = (h_{(i+1,j)}-h_{(i-1,j)})/(2*\Delta x)$

$A_7 = (\Delta x^{2}+\Delta z^{2})/ (\Delta x^{2}*\Delta z^{2})$

$A_8 = 6*\eta*u$

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